International Journal of Research in Advent Technology, Vol.6, No.12, December 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Application of Aboodh Transform for Solving Linear Volterra Integral Equations of First Kind

Sudhanshu Aggarwal[1*], Nidhi Sharma[2], Raman Chauhan[3]
[1]*Assistant Professor, Department of Mathematics, National P.G. College Barhalganj, Gorakhpur-273402, U.P., India*
[2, 3]*Assistant Professor, Department of Mathematics, Noida Institute of Engineering & Technology, Greater Noida-201306, U.P., India*
sudhanshu30187@gmail.com, 6nidhisharma@gmail.com, raman28chauhan@gmail.com

Abstract: Generally Volterra integral equations of first kind appears when we represent mathematically the advance problems of biology, chemistry, physics and engineering. In this article, we solved linear Volterra integral equations of first kind using Aboodh transform and some applications are given in application section to explain the procedure of solving linear Volterra integral equations of first kind using Aboodh transform.

Keywords: Linear Volterra integral equation of first kind, Aboodh transform, Convolution theorem, Inverse Aboodh transform.

1. INTRODUCTION

The linear Volterra integral equation of first kind is given by [1-12]

$f(x) = \int_0^x k(x,t)u(t)dt$ …………… (1)

where $u(x)$ is the unknown function, $k(x,t)$ (kernel of integral equation) and the function $f(x)$ are known real-valued functions.

In 2013, Aboodh [16] defined a new integral transform "Aboodh transform" of the function $F(t)$ as:

$$A\{F(t)\} = \frac{1}{v}\int_0^\infty F(t)e^{-vt}dt$$

$$= K(v), t \geq 0, 0 < k_1 \leq v \leq k_2$$

where the operator A is called Aboodh transform operator.

The above transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. Both these conditions are only sufficient conditions for the existence of above transform.

The application of new transform "Aboodh Transform" to partial differential equations was given by Aboodh [17]. Aboodh et al. [18] gave the connection of Aboodh transform with some famous integral transforms. Aboodh et al. [19] used Aboodh transformation method for solving delay differential equations. Aboodh et al. [20] applied Aboodh transform to solve ordinary differential equation with variable coefficients. Solution of partial integro-differential equations by using Aboodh and double Aboodh transforms methods was given by Aboodh et al [21]. Aggarwal et al. [22] discussed the application of Aboodh transform for solving linear Volterra integro-differential equations of second kind. Aggarwal et al. [23] gave the Aboodh transform of Bessel's functions. Aggarwal et al. [24] gave a new application of Aboodh transform for solving linear Volterra integral equations. Aggarwal et al. [25-26] solved linear Volterra integral equations of first kind using Mahgoub and Kamal transform in these papers. Kumar et al. [27] gave the applications of Mohand transform for solving linear Volterra integral equations of first kind. Aggarwal et al. [28] used Elzaki transform for solving linear Volterra integral equations of first kind.

The aim of this article is to determine the exact solutions for linear Volterra integral equation of first kind using Aboodh transform without large computational work.

2. LINEARITY PROPERTY OF ABOODH TRANSFORM [22-23]:

If Aboodh transform of $F(t)$ and $G(t)$ are $H(v)$ and $I(v)$ respectively then

$$A\{aF(t)+bG(t)\} = aA\{F(t)\}+bA\{G(t)\}$$

$$\Rightarrow A\{aF(t)+bG(t)\} = aH(v)+bI(v),$$

where a, b are arbitrary constants.

3. ABOODH TRANSFORM OF SOME ELEMENTARY FUNCTIONS [22-23]:

S.N.	$F(t)$	$A\{F(t)\} = K(v)$
1.	1	$\frac{1}{v^2}$
2.	t	$\frac{1}{v^3}$
3.	t^2	$\frac{2!}{v^4}$
4.	$t^n, n\epsilon N$	$\frac{n!}{v^{n+2}}$
5.	$t^n, n>-1$	$\frac{\Gamma(n+1)}{v^{n+2}}$
6.	e^{at}	$\frac{1}{v^2-av}$
7.	$sinat$	$\frac{a}{v(v^2+a^2)}$
8.	$cosat$	$\frac{1}{v^2+a^2}$
9.	$sinhat$	$\frac{a}{v(v^2-a^2)}$
10.	$coshat$	$\frac{1}{v^2-a^2}$

4. CONVOLUTION OF TWO FUNCTIONS [13-15]:

The convolution (Faltung) of two functions $F(t)$ and $G(t)$ is denoted by $F(t)*G(t)$ and it is defined as

$$F(t)*G(t) = F*G = \int_0^t F(x)G(t-x)dx$$
$$= \int_0^t F(t-x)G(x)dx$$

5. CONVOLUTION THEOREM FOR ABOODH TRANSFORMS [21-23]:

If Aboodh transform of $F(t)$ and $G(t)$ are $H(v)$ and $I(v)$ respectively then

$$A\{F(t)*G(t)\} = vA\{F(t)\}A\{G(t)\}$$
$$= vH(v)I(v)$$

6. INVERSE ABOODH TRANSFORM [22-23]:

If Aboodh transform of $F(t)$ is $K(v)$ then $F(t)$ is called the inverse Aboodh transform of $K(v)$ and mathematically it is defined as

$$F(t) = A^{-1}\{K(v)\}$$

where the operator A^{-1} is the inverse Aboodh transform operator.

7. INVERSE ABOODH TRANSFORM OF SOME ELEMENTARY FUNCTIONS [22-23]:

S.N.	$K(v)$	$F(t)=A^{-1}\{K(v)\}$
1.	$\frac{1}{v^2}$	1
2.	$\frac{1}{v^3}$	t
3.	$\frac{1}{v^4}$	$\frac{t^2}{2!}$
4.	$\frac{1}{v^{n+2}}, n\epsilon N$	$\frac{t^n}{n!}$
5.	$\frac{1}{v^{n+2}}, n>-1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{v^2-av}$	e^{at}
7.	$\frac{1}{v(v^2+a^2)}$	$\frac{sinat}{a}$
8.	$\frac{1}{v^2+a^2}$	$cosat$
9.	$\frac{1}{v(v^2-a^2)}$	$\frac{sinhat}{a}$
10.	$\frac{1}{v^2-a^2}$	$coshat$

8. ABOODH TRANSFORM OF BESSEL'S FUNCTIONS [23]:

a) Aboodh transform of Bessel's function of zero order $J_0(t)$:

$$A\{J_0(t)\}=\frac{1}{v\sqrt{(1+v^2)}}$$

b) Aboodh transform of Bessel's function of order one $J_1(t)$:

$$A\{J_1(t)\}=\frac{1}{v}-\frac{1}{\sqrt{(1+v^2)}}$$

9. ABOODH TRANSFORMS FOR LINEAR VOLTERRA INTEGRAL EQUATIONS OF FIRST KIND

In the present work, we will assume that the kernel $k(x,t)$of linear Volterra integral equation of first kind which is given by (1) is a difference kernel and it can be expressed by the difference$(x-t)$. Thus (1) can be expressed as

$$f(x)=\int_0^x k(x-t)u(t)dt \;\ldots\ldots\ldots\ldots\ldots (2)$$

Applying the Aboodh transform to both sides of(2), we have

$$A\{f(x)\}=A\{\int_0^x k(x-t)u(t)dt\}\ldots\ldots(3)$$

Using convolution theorem of Aboodh transform, we have

$$A\{f(x)\}=vA\{k(x)\}A\{u(x)\}$$

$$\Rightarrow A\{u(x)\}=\frac{1}{v}\left[\frac{A\{f(x)\}}{A\{k(x)\}}\right]\ldots\ldots\ldots\ldots(4)$$

Operating inverse Aboodh transform on both sides of(4), we have

$$u(x)=A^{-1}\left\{\frac{1}{v}\left[\frac{A\{f(x)\}}{A\{k(x)\}}\right]\right\}\ldots\ldots\ldots\ldots (5)$$

which is the required solution of (2).

10. APPLICATIONS

In this section, some applications are given to explain the procedure of solving linear Volterra integral equations of first kind using Aboodh transform.

Application: 1 Consider linear Volterra integral equation of first kind whose kernel containing exponential function

$$x = \int_0^x e^{(x-t)}\, u(t)dt \ldots\ldots\ldots\ldots (6)$$

Applying the Aboodh transform to both sides of(6), we have

$$A\{x\} = A\{\int_0^x e^{(x-t)}\, u(t)dt\} \ldots\ldots\ldots\ldots (7)$$

Using convolution theorem of Aboodh transform on (7), we have

$$\frac{1}{v^3} = vA\{e^x\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v^3} = v\left[\frac{1}{v^2 - v}\right] A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v^2} - \frac{1}{v^3} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Operating inverse Aboodh transform on both sides of(8), we have

$$u(x) = A^{-1}\left\{\frac{1}{v^2} - \frac{1}{v^3}\right\} = A^{-1}\left\{\frac{1}{v^2}\right\} - A^{-1}\left\{\frac{1}{v^3}\right\}$$

$$\Rightarrow u(x) = 1 - x \ldots\ldots\ldots\ldots\ldots (9)$$

which is the required exact solution of (6).

A. Application: 2 Consider linear Volterra integral equation of first kind whose kernel containing exponential function

$$sinx = \int_0^x e^{(x-t)} u(t)\, dt \ldots\ldots (10)$$

Applying the Aboodh transform to both sides of(10), we have

$$A\{sinx\} = A\{\int_0^x e^{(x-t)} u(t)\, dt\} \ldots\ldots\ldots\ldots (11)$$

Using convolution theorem of Aboodh transform on(11), we have

$$\frac{1}{v(v^2+1)} = vA\{e^x\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v(v^2+1)} = v\left[\frac{1}{v^2 - v}\right] A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{(v-1)}{v(1+v^2)} = \frac{1}{v^2+1} - \frac{1}{v(v^2+1)} \ldots\ldots\ldots (12)$$

Operating inverse Aboodh transform on both sides of(12), we have

$$u(x) = A^{-1}\left\{\frac{1}{v^2+1}\right\} - A^{-1}\left\{\frac{1}{v(v^2+1)}\right\}$$

$$\Rightarrow u(x) = cosx - sinx \ldots\ldots\ldots\ldots\ldots (13)$$

which is the required exact solution of (10).

B. Application: 3 Consider linear Volterra integral equation of first kind whose kernel containing Bessel function of zero order

$$sinx = \int_0^x J_0(x-t)u(t)\, dt \ldots\ldots (14)$$

Applying the Aboodh transform to both sides of(14), we have

$$A\{sinx\} = A\{\int_0^x J_0(x-t)u(t)\, dt\} \ldots. (15)$$

Using convolution theorem of Aboodh transform on(15), we have

$$\frac{1}{v(v^2+1)} = vA\{J_0(x)\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v(v^2+1)} = v\left[\frac{1}{v\sqrt{(1+v^2)}}\right] A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v\sqrt{(1+v^2)}} \ldots\ldots\ldots\ldots (16)$$

Operating inverse Aboodh transform on both sides of(16), we have

$$u(x) = A^{-1}\left\{\frac{1}{v\sqrt{(1+v^2)}}\right\} = J_0(x) \text{ ....(17)}$$

which is the required exact solution of (14).

C. **Application: 4** Consider linear Volterra integral equation of first kind whose kernel linear in the argument x and t

$$x^2 = \frac{1}{2}\int_0^x (x-t)u(t)\,dt \text{...... (18)}$$

Applying the Aboodh transform to both sides of(18), we have

$$A\{x^2\} = \frac{1}{2}A\left\{\int_0^x (x-t)u(t)\,dt\right\} \text{.... (19)}$$

Using convolution theorem of Aboodh transform on(19), we have

$$\frac{2!}{v^4} = \frac{1}{2}.vA\{x\}A\{u(x)\}$$

$$\Rightarrow \frac{2!}{v^4} = \frac{1}{2}.v\left[\frac{1}{v^3}\right]A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{4}{v^2} \text{ ... ... (20)}$$

Operating inverse Aboodh transform on both sides of(20), we have

$$u(x) = 4A^{-1}\left\{\frac{1}{v^2}\right\} = 4 \text{........... (21)}$$

which is the required exact solution of (18).

D. Application: 5 Consider linear Volterra integral equation of first kind whose kernel containing exponential function

$$x = \int_0^x e^{-(x-t)}\,u(t)dt \text{......(22)}$$

Applying the Aboodh transform to both sides of (22), we have

$$A\{x\} = A\left\{\int_0^x e^{-(x-t)}\,u(t)dt\right\} \text{.... (23)}$$

Using convolution theorem of Aboodh transform on(23), we have

$$\frac{1}{v^3} = vA\{e^{-x}\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v^3} = v\left[\frac{1}{v^2+v}\right]A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v^2} + \frac{1}{v^3} \text{............ (24)}$$

Operating inverse Aboodh transform on both sides of(24), we have

$$u(x) = A^{-1}\left\{\frac{1}{v^2} + \frac{1}{v^3}\right\} = A^{-1}\left\{\frac{1}{v^2}\right\} + A^{-1}\left\{\frac{1}{v^3}\right\}$$

$$\Rightarrow u(x) = 1 + x \text{.............. (25)}$$

which is the required exact solution of (22).

E. **Application: 6** Consider linear Volterra integral equation of first kind whose kernel contain unknown function with linear in the argument x and t

$$sinx = \int_0^x u(x-t)\,u(t)dt \text{......(26)}$$

Applying the Aboodh transform to both sides of (26), we have

$$A\{sinx\} = A\left\{\int_0^x u(x-t)\,u(t)dt\right\} \text{.... (27)}$$

Using convolution theorem of Aboodh transform on(27), we have

$$\frac{1}{v(v^2+1)} = vA\{u(x)\}A\{u(x)\}$$

$$\Rightarrow [A\{u(x)\}]^2 = \frac{1}{v^2(v^2+1)}$$

$$\Rightarrow A\{u(x)\} = \pm\frac{1}{v\sqrt{(1+v^2)}} \text{............ (28)}$$

Operating inverse Aboodh transform on both sides of(28), we have

$$u(x) = \pm A^{-1}\left\{\frac{1}{v\sqrt{(1+v^2)}}\right\}$$

$$\Rightarrow u(x) = \pm J_0(x) \ldots\ldots\ldots\ldots (29)$$

which is the required exact solution of (26).

F. **Application: 7** Consider linear Volterra integral equation of first kind with unity as a kernel

$$x = \int_0^x u(t)\,dt \ldots\ldots (30)$$

Applying the Aboodh transform to both sides of (30), we have

$$A\{x\} = A\left\{\int_0^x u(t)\,dt\right\} \ldots. (31)$$

Using convolution theorem of Aboodh transform on(31), we have

$$\frac{1}{v^3} = vA\{1\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v^3} = v.\frac{1}{v^2}.A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v^2} \ldots\ldots\ldots\ldots (32)$$

Operating inverse Aboodh transform on both sides of(32), we have

$$u(x) = A^{-1}\left\{\frac{1}{v^2}\right\} = 1 \ldots\ldots\ldots\ldots (33)$$

which is the required exact solution of (30).

G. **Application: 8** Consider linear Volterra integral equation of first kind with unity as a kernel

$$1 - J_0(x) = \int_0^x u(t)\,dt \ldots\ldots (34)$$

Applying the Aboodh transform to both sides of (34), we have

$$A\{1\} - A\{J_0(x)\} = A\left\{\int_0^x u(t)\,dt\right\} \ldots. (35)$$

Using convolution theorem of Aboodh transform on(35), we have

$$\frac{1}{v^2} - \frac{1}{v\sqrt{(1+v^2)}} = vA\{1\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v^2} - \frac{1}{v\sqrt{(1+v^2)}} = v.\frac{1}{v^2}.A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v} - \frac{1}{\sqrt{(1+v^2)}} \ldots\ldots\ldots\ldots (36)$$

Operating inverse Aboodh transform on both sides of(36), we have

$$u(x) = A^{-1}\left\{\frac{1}{v} - \frac{1}{\sqrt{(1+v^2)}}\right\} = J_1(x) \ldots\ldots (37)$$

which is the required exact solution of (34).

H. **Application: 9** Consider linear Volterra integral equation of first kind whose kernel containing Bessel function of zero order

$$J_0(x) - cosx = \int_0^x J_0(x-t)u(t)\,dt \ldots (38)$$

Applying the Aboodh transform to both sides of (38), we have

$$A\{J_0(x)\} - A\{cosx\} = A\left\{\int_0^x J_0(x-t)u(t)\,dt\right\} .. (39)$$

Using convolution theorem of Aboodh transform on(39), we have

$$\frac{1}{v\sqrt{(1+v^2)}} - \frac{1}{v^2+1} = vA\{J_0(x)\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v\sqrt{(1+v^2)}} - \frac{1}{v^2+1} = v.\frac{1}{v\sqrt{(1+v^2)}}.A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v} - \frac{1}{\sqrt{(1+v^2)}} \ldots\ldots\ldots\ldots (40)$$

Operating inverse Aboodh transform on both sides of(40), we have

$$u(x) = A^{-1}\left\{\frac{1}{v} - \frac{1}{\sqrt{(1+v^2)}}\right\} = J_1(x)\ldots\ldots(41)$$

which is the required exact solution of (38).

I. **Application: 10** Consider linear Volterra integral equation of first kind whose kernel containing Bessel function of order one

$$cosx - J_0(x) = -\int_0^x J_1(x-t)u(t)\,dt\ldots..(42)$$

Applying the Aboodh transform to both sides of (42), we have

$$A\{cosx\} - A\{J_0(x)\} = -A\int_0^x J_1(x-t)u(t)\,dt$$

$$\ldots\ldots\ldots\ldots.(43)$$

Using convolution theorem of Aboodh transform on(43), we have

$$\frac{1}{v^2+1} - \frac{1}{v\sqrt{(1+v^2)}} = -vA\{J_1(x)\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v^2+1} - \frac{1}{v\sqrt{(1+v^2)}} = -v\left[\frac{1}{v} - \frac{1}{\sqrt{(1+v^2)}}\right].A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v\sqrt{(1+v^2)}}\ldots\ldots\ldots\ldots(44)$$

Operating inverse Aboodh transform on both sides of(44), we have

$$u(x) = A^{-1}\left\{\frac{1}{v\sqrt{(1+v^2)}}\right\} = J_0(x)\ldots\ldots\ldots\ldots(45)$$

which is the required exact solution of (42).

11. CONCLUSION

In the present paper, we have successfully defined the Aboodh transform for solving linear Volterra integral equations of first kind. The given applications showed that very less computational work and a very little time needed for finding the exact solution of linear Volterra integral equations of first kind. In future, Aboodh transform can be applied for solving the system of linear Volterra integral equations.

REFERENCES

[1] Pipkin, A.C., A course on integral equations, Springer-Verlag, New York, 1991.
[2] Hochstadt, H., Integral equations, Wiley, New York, 1973.
[3] Kanwal, R.P., Linear integral equations, Academic Press, New York, 1971.
[4] Lovitt, W.V., Linear integral equations, Dover, New York, 1950.
[5] Tricomi, F.G., Integral equations, Interscience, New York, 1957.
[6] Wazwaz, A.M., A first course in integral equations, World Scientific, Singapore, 1997.
[7] Bitsadze, A.V., Integral equations of first kind, World Scientific, Singapore.
[8] Polyanin, A.D. and Manzhirov, A.V., Handbook of integral equations, Chapman & Hall/CRC, 2008.
[9] Moiseiwitsch, B.L., Integral equations, Dover Publications, New York, 2005.
[10] Hackbusch, W., Integral equations: Theory and numerical treatment, Birkhauser Verlag, Berlin, 1995.
[11] Linz, P., Analytical and numerical methods for Volterra equations, SIAM, Philadelphia, 1984.
[12] Jerri, A. J., Introduction to integral equations with applications, Wiley, New York, 1999.
[13] Raisinghania, M.D., Advanced differential equations, S.Chand & Company, New-Delhi, 2015.
[14] Raisinghania, M.D., Integral equations and boundary value problems, S.Chand & Company, New-Delhi, 2017.
[15] Lokenath Debnath and Bhatta, D., Integral transforms and their applications, Second edition, Chapman & Hall/CRC, 2006.

[16] Aboodh, K.S., The new integral transform "Aboodh Transform", Global Journal of Pure and Applied Mathematics, 9(1), pp. 35-43, 2013.
[17] Aboodh, K.S., Application of new transform "Aboodh Transform" to partial differential equations, Global Journal of Pure and Applied Mathematics, 10(2), pp. 249-254, 2014.
[18] Aboodh, K.S., Idris Abdullahi and Nuruddeen, R.I., On the Aboodh transform connections with some famous integral transforms, International Journal of Engineering and Information Systems, 1(9), pp. 143-151, 2017.
[19] Aboodh, K.S., Farah, R.A., Almardy, I.A. and Osman, A.K., Solving delay differential equations by Aboodh transformation method, International Journal of Applied Mathematics & Statistical Sciences, 7(2), pp. 55-64, 2018.
[20] Mohand, D., Aboodh, K.S. and Abdelbagy, A., On the solution of ordinary differential equation with variable coefficients using Aboodh transform, Advances in Theoretical and Applied Mathematics, 11(4), pp. 383-389, 2016.
[21] Aboodh, K.S., Farah, R.A., Almardy, I.A. and Almostafa, F.A., Solution of partial integro-differential equations by using Aboodh and double Aboodh transforms methods, Global Journal of Pure and Applied Mathematics, 13(8), pp. 4347-4360, 2016.
[22] Aggarwal, S., Sharma, N. and Chauhan, R., Application of Aboodh transform for solving linear Volterra integro-differential equations of second kind, International Journal of Research in Advent Technology, 6(6), pp. 1186-1190, 2018.
[23] Aggarwal, S., Gupta, A.R. and Agrawal, D., Aboodh transform of Bessel's functions, Journal of Advanced Research in Applied Mathematics and Statistics, 3(3), pp. 1-5, 2018.
[24] Aggarwal, S., Sharma, N. and Chauhan, R., A new application of Aboodh transform for solving linear Volterra integral equations, Asian Resonance, 7(3), pp. 156-158, 2018.
[25] Aggarwal, S., Sharma, N. and Chauhan, R., Application of Mahgoub transform for solving linear Volterra integral equations of first kind, Global Journal of Engineering Science and Researches, 5(9), pp. 154-161, 2018.
[26] Aggarwal, S., Sharma, N. and Chauhan, R., Application of Kamal transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(8), pp. 2081-2088, 2018.
[27] Kumar, P.S., Saranya, C., Gnanavel, M.G. and Viswanathan, A., Applications of Mohand transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(10), pp. 2786-2789, 2018.
[28] Aggarwal, S., Chauhan, R. and Sharma, N., Application of Elzaki transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(12), pp. 3687-3692, 2018.

Sudhanshu Aggarwal received his M.Sc. degree from M.S. College, Saharanpur in 2007. He has also qualified CSIR NET examination (June-2010, June-2012, June-2013, June-2014 & June-2015) in Mathematical Sciences. He is working as an Assistant Professor in National P.G. College Barhalganj Gorakhpur. He is equipped with an extraordinary caliber and appreciable academic potency. He has around ten years of teaching experience at various engineering colleges affiliated to AKTU. His fields of interest include Integral Transform Methods, Differential and Partial Differential Equations, Integral Equations and Number Theory. He has published many research papers in national and international journals.

Nidhi Sharma received her M.Sc. degree from Banasthali University Rajasthan in 2015. She is working as an Assistant Professor in Noida Institute of Engineering & Technology, Greater Noida since 2015. She has also qualified IIT Jam exam (2013). She has published many research papers in national and international journals. Her fields of interest include Optimization, Fuzzy Logic and Topological Spaces.

Raman Chauhan has completed M.Sc. (Mathematics) from Indian Institute of Technology, Roorkee in 2011. He is working at NIET, Greater Noida since 2013 as a dynamic, successful and responsible faculty member in the department of Mathematics. He has also qualified IIT Jam exam (2009) and Gate exam (2011, 2013, 2014 & 2015). His fields of interest include Optimization, Integral Equations and Abstract Algebra.